

**GENDER AND MATHEMATICS:
WHAT IS KNOWN AND WHAT DO I WISH WAS KNOWN?**

Elizabeth Fennema
Wisconsin Center for Education Research

May 2000

Prepared for the Fifth Annual Forum of the National Institute for Science Education, May 22-23, 2000,
Detroit, Michigan.

COMPLEXITY! After studying issues related to gender and mathematics for about 3 decades, I have decided complexity most adequately describes what I know. It also is a major component of the questions that I wish I knew the answers to. Gender differences in learning mathematics are complex; the multiplicity of forces and environments that operate within our Society to influence that learning are complex; it is complex to design effective intervention programs; the role that biological factors might or might not play are complex; it is certainly complex to conduct research about gender and mathematics; it is even more complex to interpret research for practitioners. While research over the past three decades has made significant contributions to both defining and understanding this complexity, there is much left to know. What I am trying to do in this paper is share with you what I have come to know and understand through my study of gender and mathematics. To do this, I rely on my knowledge and understanding of extant scholarship, societal forces, and personal beliefs as well as my perception of how all of these interact to influence the development and maintenance of gender differences in mathematics.

Research and Personal Beliefs

All scholarship is strongly influenced by personal beliefs. Sometimes beliefs are overt and explicitly defined, but in many cases the beliefs are covert and not easily recognized. In the instance of scholarship related to gender, beliefs exert a strong influence. Gender is a vital part of each human being that cannot be ignored. Although I once wrote about the necessity for teachers to become sex-blind, part of my journey has led me to know that is impossible. We cannot eliminate our sex-role identity from our learning, our beliefs, or our scholarship.

Some definitions would be helpful because words communicate beliefs. Two different words have been used that imply beliefs about *causation* of differences between females and males, i.e. *sex* and *gender*. And my own writing, as well as the writing within the field, reflects scholars' maturing view of the complexity of causation of differences between females and males. Work published before about 1970 used the phrase *sex* differences when research results were reported. This phrase contributed to the implication that any found differences were biologically, and thus genetically, determined. According to this belief, these differences were immutable and could not be changed. Therefore, schools could accept the difference as non-changeable and not work to change them. Work published during the 70s and 80s often used the term *sex-related* differences that many hoped would be interpreted as an indication that while the behavior of concern was clearly related to the sex of the subjects, it was not necessarily genetically determined. More recently most scholars have discussed *gender* differences believing that such a term has a stronger flavor of social or environment causation of differences that are observed between the sexes. While the meaning and use of words is a murky area, when I use the word *sex*, I am referring to biologically determined behaviors. When I use the word *gender* I am inferring social or environmentally causation of behaviors that differ for females and males. Of course, it is impossible to totally separate social and biological influences and perhaps it isn't always necessary. However, I shall try to be consistent even though I am sure that my mixing of the two words will reflect the complexity of sorting out causation of learning differences between males and females.

My own value position, or set of personal beliefs, has strongly influenced what I have done and what I say here. It is important (to me at least) that I make some of my beliefs explicit. I have always believed that through scholarly activity, I can learn how to better facilitate the learning of mathematics by females and males. Coupled with this is my belief that classrooms play a major role in determining what an individual learns. It follows that if one identifiable group of people is not learning as well as another group, then the educational environment can and should be modified to ensure that these group differences are eliminated. Another of my beliefs that seems to be particularly relevant to this paper has to do with the place of mathematics in education. I believe that all pre-university students should learn mathematics, not just for the sake of learning one of the most important bodies of knowledge that humans have developed, but because mathematical knowledge provides power in understanding the world as well as the possibility of choice. Without learning mathematics, one cannot choose to pursue graduate study in many fields, change careers, or do many other things. Not all people will choose to do careers where the knowledge of mathematics is essential, but I believe they should have the option to make that choice. My

entire professional career has been predicated on the over-arching belief that women deserve equity with men in all walks of life, and that belief has been reflected in a significant part of my scholarly activities in the area of gender and mathematics since all of my intersecting beliefs can be easily seen in my scholarship. Does this mean that I am not a very good scholar or only report what I want to believe? Not at all! It just means that the questions that I chose to investigate and the methods I chose to use were strongly influenced by those beliefs. (For a more complete discussion of this point of view, see Lagemann & Shulman, 1999.)

One of my original, naïve ideas had to do with what *equity* is. While it appeared easy to define, this has not proved to be the case. Does equity mean that females and males should have an equal opportunity to learn whether or not they avail themselves of that opportunity? This is the definition adopted in much federal legislation that has dealt with equity such as Title IX. No one can restrict access to mathematics courses on the basis of sex. Both girls and boys should be able to enroll in the same mathematics courses, textbooks should portray males and females in identical roles, girls and boys should have equal access to computers, etc. While these overt things have been fairly well accomplished, this definition of equity has not been achieved as can be seen in the various studies that looked at teacher-pupil interactions. Researchers have gone into classrooms and counted the number of times teachers interact with boys and girls and attempted to document well defined educational experiences. For example, how many times do teachers call on students of each sex to solve mathematical problems, praise them, etc. I know of no study that does not indicate that teachers interact with boys more than with girls, so there is not equality of educational experiences.

Instead of equal educational experiences, equity could mean *equality of outcomes*, i.e. that females should learn exactly the same mathematics as do males, be able to perform the same on various measures of mathematical learning, and have the same personal feelings toward oneself and mathematics. Under this definition when equity is achieved, girls will be as confident about learning mathematics as are boys, girls will believe that they have as much control of their mathematics learning as do boys, etc., and there would be no differences found on such tests as the SAT or local, state, national, or international measures of achievement. While other papers will address gender differences in attitudes toward and learning mathematics and so I won't be expansive about it, I read the literature to indicate that whenever higher level cognitive skills are measured, girls are still not performing as well as boys, nor do they hold as positive an attitude toward mathematics.

Some have suggested that equality of outcomes can be achieved by different instructional methods for females and males. It is based on the belief that males and females learn in different, but equitable ways. E.g., some research suggests that girls learn better in cooperative groups and boys learn better in competitive groups, or that single-sex classrooms adapted for a female learning style should be established. This particular definition has received a great deal of scholarly and media attention in the last few years. (Most of the Gilligan work is based on it.) While the implied definition of equity in the mathematics *Standards* directly rejects this definition when it says that "All students should learn mathematics," it then goes ahead to recommend one type of mathematics and instructional procedures for all groups.

Others have suggested that equity has to do with social justice for all in learning mathematics. (And that seems to me just to bring in another poorly understood term.) As can readily be seen, merely understanding the word *equity* is complex. (See Secada, 1989; and Leder, Forgasz, & Solar, 1996 for an expanded discussion of these ideas.) There is no correct definition or understanding of what it is. Certainly research won't help much in resolving it. While various kinds of philosophical discussion can enrich our understanding, defining equity comes down to what one personally believes.

What I believe is that equity in mathematics education will be achieved when there are no perceivable differences between the mathematics known, or how females and males feel about themselves and mathematics. If in order to achieve this goal, it is necessary to have separate instructional methods, they are acceptable. If teachers have to treat boys and girls identically, they can be trained to do so. I believe

that equity using such a definition means that equity is achievable. It also follows logically that research will help in understanding how it can be achieved as well as providing educational guidelines for achieving it.

Research from 1970-1990

During the years between 1970 and 1990, there were probably more research studies published concerned with gender and mathematics than in any other area (Leder, 1996). This work has been well reviewed elsewhere and I won't duplicate these reports. To give a flavor of the work, I will briefly review my own work that was done in association with a variety of colleagues. In 1974, my first article about gender, a review of extant work on sex difference in mathematics, was published in the *Journal of Research in Mathematics Education*. In this article, I concluded that while many studies had been poorly analyzed and/or included sexist interpretations, there was evidence to support the idea that there were differences between girls' and boys' learning of mathematics, particularly in activities that required complex reasoning; that the differences increased at about the onset of adolescence; and that these differences were recognized by many leading mathematics educators. As an aside, it was really the writing of that 1974 article that turned me into an active feminist. It compelled me to recognize the bias that existed towards females, which was exemplified by the recognition and acceptance by the mathematics education community at large of gender differences in mathematics as legitimate.

The Fennema-Sherman studies (Fennema & Sherman, 1977, 1978; Sherman & Fennema, 1977), sponsored by the National Science Foundation and published in the mid 1970s, documented sex-related differences in achievement and participation in Grades 6 to 12. Although there were many subtle results, these findings basically agreed with those of my original review. In addition, Sherman and I found gender differences in the election of advanced level mathematics courses. When we coupled the achievement differences with the differential course enrollment, we hypothesized that if females participated in advanced mathematics classes at the same rate that males did, gender differences would disappear. Many things are learned as one does research, and from the stating of this hypothesis, I learned that what you write and say stays with you a long time. This hypothesis, labeled by others as the *differential course-taking hypothesis*, became a point of attack by Julian Stanley and Camilla Benbow (Stanley & Benbow, 1980), who used interpretations of some of their studies as a refutation of our hypothesis. They went ahead to use their work as evidence that gender differences in mathematics are genetic. Although widely attacked and disproved, the publication of their claims in the public media did have unfortunate repercussions (Jacobs & Eccles, 1985).

Affective or attitudinal variables were also examined in the Fennema-Sherman studies. Identified as critical were beliefs about the usefulness of mathematics and confidence in learning mathematics, with males providing evidence that they were more confident about learning mathematics than were females, and males believing that mathematics was, and would be, more useful to them than did females. It also became clear that while young men did not strongly stereotype mathematics as a male domain, they did believe much more strongly than did young women that mathematics was more appropriate for males than for females. The importance of these variables, their long-term influence, and their differential impact on females and males was reconfirmed in many of our later studies, as well as by the work of many others (Leder, 1992).

One cognitive variable also studied in the Fennema-Sherman studies was spatial skills or spatial visualization, which I continued to investigate in a three-year longitudinal study in collaboration with Lindsay Tartre (Fennema & Tartre, 1985). Differences between females and males in spatial skills, particularly spatial visualization or the ability to visualize movements of geometric figures in one's mind, had long been reported (Maccoby & Jacklin, 1974). Since items that measure spatial visualization are so logically related to mathematics, it has always appeared reasonable to believe that spatial skills contributed to gender differences in mathematics. We found that while spatial visualization was positively correlated with mathematics achievement (that does not indicate causation), not all girls are handicapped by inadequate spatial skills, but perhaps only those girls who score very low on spatial tasks.

Although they were not particularly innovative nor did they offer insights that others were not suggesting, the Fennema-Sherman studies had a major impact due to a variety of reasons. They were published in highly accessible journals just when the concern with gender and mathematics was growing internationally. Partly because the studies were accessible, not generally controversial, and because they employed fairly traditional methodology, their findings have been accepted by the community at large. The studies were identified by two independent groups (Walberg & Haertel, 1992; Anonymous, in preparation) as among the most often quoted social science and educational research studies during the 80s and 90s. Each week, I still receive at least one request for information about the Fennema-Sherman Mathematics Attitude Scales that were developed for those studies. The problems of gender and mathematics were defined and documented in terms of the study of advanced mathematics courses, the learning of mathematics, and certain related variables that appeared relevant both to students' selection of courses and learning of mathematics. Many have used them as guidelines for planning interventions and other research.

After completing the Fennema-Sherman studies, with the indispensable aid of many others (Laurie Reyes Hart, Peter Kloosterman, Mary Koehler, Margaret Meyer, Penelope Peterson, and Lindsay Tartre), I broadened my area of investigation to include other educational variables, particularly teachers, classrooms, and classroom organizations. We studied teacher-student interactions, teacher and student behaviors, and characteristics of classrooms and teaching behaviors that have been believed to facilitate females' learning of mathematics.

The series of studies dealing with educational variables, reported and summarized in the book edited by Gilah Leder and me (1990), suggested that many studies have documented that it is relatively easy to identify differential teacher interactions with girls and boys. In particular, teachers interact more with boys than with girls, praise and scold boys more than girls, and call on boys more than girls. However, the impact of this differential treatment is unclear and difficult to ascertain. The data that resulted from the studies reported in this book do not support the premise that all differential teacher treatment of boys and girls is very closely related to gender differences in mathematics. (See also Eccles & Blumenfeld, 1985; Koehler, 1990; Leder, 1982).

However, when more subtle examples of teacher-pupil behavior were studied, Peterson and I (Fennema & Peterson, 1986; Peterson & Fennema, 1985) found that small differences in teacher behavior combined with the organization of instruction, made up a pattern of classroom organization that appeared to favor males. For example, competitive activities encouraged boys' learning and had a negative influence on girls' learning, while the opposite was true of cooperative learning. Since competitive activities were much more prevalent than cooperative activities, it appeared that classrooms we studied were more often favorable to boys' learning than to girls' learning.

In connection with this series of studies, Peterson and I proposed the Autonomous Learning Behaviors model, which suggested that because of societal influences (of which teachers and classrooms were main components) and personal belief systems (lowered confidence, attributional style, belief in usefulness), females do not participate in learning activities that enable them to become independent learners of mathematics (Fennema & Peterson, 1985). This model still appears valid, although my understanding of what independence is has grown, and I believe that independence in mathematical thinking may be learned through working in cooperation with others to solve mathematical problems.

Identifying behaviors in classrooms that influence gender differences in learning and patterns has been difficult. Factors that many believed to be self-evident have not been shown to be particularly important, and I do not believe that we have sufficient evidence that would allow us to conclude that teachers interacting more or differently with girls than with boys is a major contributor to the development of gender differences in mathematics. Many intervention programs have been designed to help teachers recognize how they treat boys and girls differently. Unfortunately, such programs do not appear to have been successful in eliminating gender differences in mathematics. I believe that differential teacher treatment of boys and girls is merely one piece of the complexity of the causes of gender differences in

mathematics.

My next set of studies was conducted with Janet Hyde. For these studies, we did a series of meta-analyses of extant work on gender differences reported in the US, Australia, and Canada (Hyde, Fennema, & Lamon, 1990; Hyde, Fennema, Ryan, & Frost, 1990). These results indicated that while gender differences in mathematics achievement might have decreased, they still existed in tasks that required functioning at high cognitive levels. It also seemed that when tests measured problem solving at the most complex cognitive level, the more apt there were to be gender differences in mathematics in favor of males. The international assessment reported by Gila Hanna (Hanna, 1989) showed results that basically confirmed this.

My work has not been the sole chain of inquiry that has occurred during the last two decades. The work of Jacquelynne Eccles, Gilah Leder (Leder, 1992), and others has been conducted independently and closely parallels the major themes I have addressed. An overly simplistic and not inclusive summary suggests that during this time, scholars documented that differential mathematics achievement and participation of females and males existed; some related educational and psychological variables were identified; explanatory models were then proposed; and finally (or concurrently in some cases) interventions, based on the identified variables, were designed to alleviate the documented differences.

One line of inquiry that I have not pursued, but that adds a significant dimension and more complexity to the study of gender and mathematics, is the work that has divided the universe of females into smaller groups. In particular, the work of the High School and Beyond Project (a large multi-year project that documented gender differences in mathematics as well as many other areas) as interpreted by Secada (1992), and the work of Reyes (Hart) and Stanic (1988) have investigated how socio-economic status and ethnicity interact with gender to influence mathematics learning. The US, as many other countries, is a highly heterogeneous society, made up of many layers, divisions, and cultures. The pattern of female differences in mathematics varies across these layers and must be considered.

Intervention Studies

By about 1980, there were some rather consistent findings that described gender differences in mathematics. Based on these findings and with the help of three others (Joan Daniels Pedro, Patricia Wolleat, and Ann Becker DeVaney), I developed an intervention program called *Multiplying Options and Subtracting Bias* (Fennema, Wolleat, Becker, & Pedro, 1980). This program lasted approximately one-hour and was focused on parents, teachers, counselors, boys, and girls. It was composed of sharply focused videotapes to be viewed by the target group, and a workshop guide that included suggestions for follow-up activities. It was extensively evaluated, particularly with regard to its effectiveness in increasing girls' participation in advanced secondary school mathematics classes and its impact on confidence and perceived usefulness. We found that this short intervention that helped the target audiences recognize the importance of mathematics and the stereotyping of mathematics that was prevalent, resulted in more girls, and more boys, electing to take mathematics courses. A more negative finding, however, was that pointing out the sexism that existed in classrooms and environment increased the girls' anxiety about mathematics.

Since *Multiplying Options and Subtracting Bias* was completed a plethora of other intervention programs have been designed and implemented. The *Women in Science* program of the National Science Foundation funded 136 projects between 1976 and 1981, and *The Women in Engineering Programs* reported 395 interventions in 1975 and 859 in 1991 (Leder, Forgasz, & Solar, 1996). Many other programs have been developed by other groups that range from short-term interventions of one day or less to long-term programs that focus on curriculum and instructional change, total school re-organization (changing from coed to single sex classrooms), or teacher education. The population targeted has included many diverse groups: teachers, university faculty that work with pre-service teachers, parents, females in general or specific groups of females such as those who are traditionally disadvantaged. Sometimes boys are the focus. Pre-school to graduate programs have been developed. The content of the

interventions has ranged from curriculum content to hands on activities illustrative of what users of mathematics do.

While some of these programs have been evaluated, the effectiveness of many of the programs has not been well documented. Several excellent summaries are available and I urge reading of them for more information (see for example Campbell; Clewell and her colleagues; or Gilah Leder and her colleagues, 1996). Even without specific evidence of total effectiveness with the various targeting populations, it is accurate to say that the programs have been extremely effective in educating the general public about the importance of learning mathematics to both boys and girls. Unfortunately, the message that has been received by many is that “girls can not do mathematics” (Eccles, 1985).

In summary, because I was asked so often to speak about gender and mathematics, I compiled yearly lists of what I had concluded that research had shown. Following is a portion of the list I made in 1990.

Gender Differences in Mathematics: 1990

1. Gender differences in mathematics may be decreasing.
2. Gender differences in mathematics still exist in:
 - learning of complex mathematics
 - personal beliefs in mathematics
 - career choice that involves mathematics
3. Gender differences in mathematics vary:
 - by socioeconomic status and ethnicity
 - by school
 - by teacher
4. Teachers tend to structure their classrooms to favor male learning.
5. Interventions *can* move towards achieving equity in mathematics.

On the basis of an examination of the five items on this list, it is clear that in the years following my original review in 1974, my understanding of gender and mathematics had grown as far as related variables were concerned, but the same gender differences, albeit perhaps smaller, still existed. I could describe the problem more precisely. I knew that large variations between groups of females existed; I knew that there were differences among schools and teachers with respect to gender and mathematics issues; I knew that females and males differed with respect to personal beliefs about mathematics; and I knew that some interventions could make a difference. I was increasing my understanding that the issue of gender and mathematics was extremely complex.

Now before I sound too pessimistic, it should be noted that there were many females who were achieving in mathematics and are currently pursuing mathematics-related careers. However, let me reiterate that in spite of some indications that achievement differences were becoming smaller—and they were never very large anyway—they still existed in those areas involving the most complex mathematical tasks. These differences became more evident as students progressed to middle and secondary schools. There were

also major differences in participation in mathematics-related careers. Many women, capable of learning the mathematics required, chose to limit their options by not studying mathematics. And while I have no direct data, I strongly suspected that the learning and participation of many women, who might be in the lower two-thirds of the achievement distribution, have not progressed at all. I had to conclude that many of the differences that were reported in the 1970s, while smaller overall than they were then, still existed in 1990.

The 1990s and Beyond

My personal odyssey with gender and mathematics has continued until the present day. Although I took a hiatus that resulted in two changes of direction. While I continued to accept without question the basic premise of the International Commission for Mathematics Instruction Study Conference (1992) that “there is no physical or intellectual barrier to the participation of women in mathematics,” about 1990 I stopped doing any original work on gender and mathematics. I had worked hard for about 25 years, but in spite of all that work and the additional work done by many dedicated educators, mathematicians, and others, the *problem* still existed in much the same form that it did in 1974. Not only was I discouraged, but I was convinced a new perspective on the research about women, girls, and mathematics was needed. Fortunately, I was not alone in recognizing that research on gender and education needed to change. And direction for the change came from two directions: cognitive science and feminist scholarship.

Cognitive Science and its Impact

One of the emerging and productive educational research paradigms has been cognitive science, or the “scientific study of mental events, primarily concerned with the contents of the human mind, . . . and the mental processes in which people engage” (Brown & Borke, 1992). One major assumption underlying much of cognitive science research is that the principles of cognitive development, particularly in mathematics, can be applied universally to all people, both females and males. And indeed, findings that indicate the universality of the emerging theories of cognitive development have been reported from a number of countries and cultures. (See for example, Adetula, 1989; Olivier, Murray, & Human, 1990; Secada, 1991.)

Within the mathematics education research community, cognitive science methodologies have been used to investigate both students’ and teachers’ thinking. Often when students are studied, they are asked to report mental strategies they have used to solve various kinds of mathematical problems derived from a precise definition of a mathematical domain. Particularly in the elementary school, the robust body of knowledge about children’s thinking in arithmetic has had major impact on the instructional program. Effective professional development programs have also been built on knowledge derived from cognitive science studies. These programs usually help teachers to understand their students’ thinking and to build their instructional programs on what the students know. (For example see Carpenter, Fennema, Franke et al., 1999).

The question of gender differences doesn’t appear to have been interesting to cognitive science scholars, perhaps because they believed that the patterns of mental activity they were finding were universal and thus there were no gender differences in cognitive behaviors to be found. But, there have been a few studies that have indicated that the assumption of no gender differences in mathematical thinking may not be true. Carr and Jessup (1997) reported that girls in Grades 1-3 tended to use observable or overt strategies such as counting, while the same age boys tended to use mental strategies. Gallagher and DeLisi (1994) reported that while there were no overall differences in the number of selected SAT items answered by females or males, females tended to use more conventional or commonly taught strategies, while males tended to use more untaught strategies. Some working within this paradigm have chosen not to report the gender differences that they found (Reys, Rybolt, Bestgen, & Wyatt, 1982), “perhaps because they were unable to provide any reasons for their finding, and they decided to put it aside” (Sowder, 1998, p. 12). As a result, there are not many studies related to gender that have been done using a cognitive science perspective.

Once again, I turn to my own and my colleagues' work, most of which has come from the Cognitively Guided Instruction Project (CGI). CGI is a professional development program designed to help teachers understand their student's mathematical thinking and to use this understanding to design instruction. The program development and related research was supported by the National Science Foundation for about 10 years and resulted in many studies that focused on teachers, instruction, and students' learning when they had been in CGI classrooms. Overall, the results indicated that teachers could learn to accurately assess their children's thinking using some cognitive science methodologies. When the teachers gathered knowledge about their students' strategies for solving mathematics problems, they modified their instruction rather dramatically so that their students' knowledge and mental processes became a significant part of the instructional programs. Students who had learned in CGI classrooms learned significantly more than their non-CGI counterparts.

One study investigated teachers' knowledge of and beliefs about boys' and girls' success in mathematics (Fennema, Peterson, Carpenter, & Lubinski, 1990). Although teachers thought the attributes of girls and boys who succeeded in mathematics were basically similar, teachers' knowledge about which boys were successful was more accurate than teachers' knowledge about which girls were successful, and teachers attributed the boys' successes more to ability and girls' successes more to effort. Linda Weisbeck's (1992) results add some interesting dimensions to our knowledge of teachers' cognitions. During stimulated-recall interviews, teachers reported that they thought more about boys than about girls during instruction. However, the characteristics they used to describe girls and boys were very similar.

It appears that teachers were very aware of whether the child they were interacting with was a boy or a girl. However, they didn't think that there were important differences between girls and boys that should be attended to as they made instructional decisions. Boys just appeared to be more salient in the teachers' minds. Teachers appeared to react to pressure from students, and they got more pressure from boys. Interventions designed with this finding in mind would be very different from interventions that assume that teachers are sexist.

What Cognitive Science has Taught Us about Girls' and Boys' Mathematics

One extensive study, Cognitively Guided Instruction (CGI), was done by Tom Carpenter, me, and several others (Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). In a 3-year longitudinal study we studied teachers and their students as they progressed from Grade 1 through Grade 3 (Fennema, Carpenter, et al., 1998). Once or twice each year, children who had learned their mathematics in CGI classrooms, were asked to solve a variety of problems (number facts, addition/subtraction word problems, non-routine, and extension problems) and to report how they solved the problems. We found no gender differences in correctly solving number fact, addition/subtraction, or non-routine problems throughout the three years of the study. This finding was in agreement with literature where it has been widely reported, as well as believed, that gender differences do not emerge until early adolescence. In our study, however, each year from Grade 1 to Grade 3 we found strong and consistent gender differences in the *strategies* used to solve problems, with girls tending to use more concrete strategies like modeling and counting and boys tending to use more abstract strategies that reflect conceptual understanding. In other words, the mental processing of boys and girls were different, and we also found some significant achievement differences in solving extension problems.

By the end of the third grade, the girls used more standard algorithms than did the boys. On the problems that required flexibility in extending one's problem solving procedures, boys were more successful than were girls. The ability to solve the extension problems in the third grade appeared to be related to the use of invented rather than procedural algorithms in earlier grades, as both girls and boys who had used invented algorithms early were better able to solve the extension problems than those who had not.

Because these results were so unexpected to us, we asked 3 prominent scholars who had worked in different areas to interpret the results and to speculate about the results' importance, causation, and potential impact on future mathematical learning. (See *Educational Researcher*, 1998, 27, pp. 4-22).

While one scholar was somewhat skeptical that the results were large enough to be important, the others felt that they were critically important, and might presage the gender differences that are found to increase as students move into advanced mathematics. The importance of the findings was reflected in Judith Sowder's words:

Children who can invent strategies for computational tasks show a more advanced grasp of basic mathematical concepts than those children who are dependent on (other strategies). The children who can invent strategies are more likely to find sense in the mathematics they are learning and come to believe that mathematics makes sense and to seek out sense in the mathematics they continue to learn. Their understanding will lead to deeper confidence in their ability to do mathematics. *They have a better chance of succeeding mathematically.* (p. 13) (Italics added)

The mathematical understanding that was indicated by the strategies used more by the boys' than the girls' is important for development of fundamental concepts and students' ability to be flexible in new situations. Thus, the more abstract strategies that children invent to solve various problems is probably related to their future understanding of mathematics, and could indeed help to explain the gender differences in older learners that had been evident for many decades. Major gender differences in performance usually don't appear until sometime in adolescence when they are more often exhibited in complex mathematics tasks, particularly on tests of problem solving. The gender differences that were reported in this study strongly suggest that more girls than boys were following a pattern of mathematical development and learning that was not based on understanding. And the lack of understanding becomes more critical as students progress through school. While it is possible to learn to do arithmetic procedures in the early grades without understanding, it becomes more and more difficult to learn advanced ideas unless a foundation of understanding is present from the very beginning.

Overall this study suggests that gender differences appear earlier and are more complex than had previously been recognized. The results certainly call into question an assumption that is prevalent in the various recommendations for reform in mathematics teaching and learning. It is widely believed that one reformed curriculum with its accompanying instructional design and methodology will suffice for all children. However, it seems to me that the results of this study suggest that without explicit attention to traditionally underachieving groups, all groups of children will **not** learn mathematics equitably. Many have identified classrooms such as the ones in which these children were learning as epitomizing needed reforms in mathematics teaching. These CGI classrooms emphasized complex mathematical tasks (problem solving), communication about mathematics, and learning with understanding—all of which are major tenets of mathematics education reform. And it is clear that the students who learned their mathematics in these classrooms did learn and understand significantly more than did children in more traditional classrooms, but there were still dramatic gender differences.

Many advocates of basing curriculum on understanding as well as most scholars who study teaching and learning believe that equity issues can be addressed by improving mathematics instruction for all. (See for example Carey, Fennema, Carpenter, & Franke, 1995). Pervasive in the mathematics education community there is an underlying assumption that one program based on understanding will enable all students to learn in an equitable fashion. Based on this study, *this assumption may not be valid*. The teachers in this study were participating in a professional development program that emphasized reform ideas and included discussion about equity issues. Much of the instruction approached the ideal of furthering children's understanding in a way that is consistent with current recommendations. We did intense observations in each classroom and saw no evidence of teachers treating girls and boys differently. Yet, strong gender differences were manifested by children who had learned in these classrooms for three years. While the differences found were probably no greater and could have been less than what exists in traditional classes, the fact remains that they existed. And most would agree that these differences could lead to stronger differences in the future. The results call into question the suggestion of the *Standards* (National Council of Teachers of Mathematics, 1989, 1991) that major curriculum reform, by itself, will provide equity in mathematics education.

Explanations of why boys, more than girls, developed mathematical understanding as they moved through Grades 1-3 can only be speculative. All the scholars believed that something was taking place in the classrooms that encouraged these gender differences to emerge. It was addressed most directly by Hyde and Jaffee who are social and feminist psychologists. They suggested that the differences were a result of differential treatment of girls and boys by the teachers (Hyde & Jaffee, 1998). They suggested that because the type of mathematical program used in the CGI classrooms was based on teachers having the freedom to make instructional decisions, the teachers' stereotypical beliefs about gender and mathematics could have led them to interact differently with boys and girls, and that in turn led to the differences found.

Another hypothesis has to do with the children's choice of strategies to report. Children in these classrooms had a great deal of freedom in deciding how to solve problems and also in deciding what strategies to report about their problem solutions. It is clear that children often have a variety of strategies to use to solve problems, and strategy use is a matter of preference. For some reason, did the girls prefer to use and report strategies that would have an influence on the development of their understanding? This choice may have inhibited the development of fluency with more abstract strategies. Hyde and Jaffee inferred that this may have been so and suggest that the freedom to choose may have permitted the children's stereotypical beliefs to influence strategy use and thus the development of understanding in these classrooms.

Perhaps girls chose to use strategies that could make their ideas clear (e.g., modeling with concrete materials) partly because the teachers and peers wanted to understand each child's thinking. Hyde and Jaffee suggested that girls, more than boys, are more socially aware of others' responses and are considerate of others' needs and/or are more compliant. It was a basic tenet of these classrooms that the teacher's understanding of each child's thinking was essential. It was obvious in the classrooms that teachers wanted to know what the child had done, and children were equally eager to make sure that the teachers understood. Modeling strategies are easier to report and to understand than are invented algorithms. In order to comply with teachers' wishes or to help them and other students to understand their responses, did girls tend to report modeling strategies rather than other kinds? Or did the girls simply prefer reporting the less abstract strategies? Sowder supported this idea when she suggested that although the girls may have understood invented strategies as well as the boys did, they may have just preferred less abstract strategies. Many believe that student preferences are important, but in this case, it may have been that using such strategies inhibited the development of more abstract strategies.

In summary, it is clear that cognitive science methodologies are providing tools for us to gain deeper understanding of the complexity of gender differences. We are just beginning to understand differences in mental activities between girls and boys and to assess their impact on learning. We also know that teachers' thoughts about girls and boys influence their instructional decisions. Understanding teachers' beliefs and knowledge about girls and boys will provide important information as we plan interventions to achieve equity.

Feminist Perspectives

While it is impossible to expound here very deeply on various feminists theories that are being used to shape research, it is safe to say that these theories are influencing many people's world-view. I am no expert on feminist theories and their accompanying research paradigms, but it seems to me that people working in feminist perspectives share one common component. Without exception, they focus on interpreting the world and its components from a female's point of view, and the resulting interpretations are dramatically different from world-views that used to be accepted.

Feminist scholars argue very convincingly that most of our beliefs, perceptions, and scholarship, including most of our scientific methodologies and findings, have been and are dominated by male perspectives or interpreted through masculine eyes. According to feminist scholars, this perspective has resulted in a view of the world that is incomplete at best and often wrong. If females' actions and points

of view had been considered over the last few centuries, according to many of these feminist scholars, our perceptions of life would be much different today. A basic assumption of feminist work is that there are basic differences between females and males that are more prevalent than the obvious biological ones. These differences result in males and females interpreting the world differently. Many of these scholars present convincing arguments about how the world influences males and females differently. It appears irrelevant whether these differences are inherent or environmentally caused, and most feminist writers that I have read are basically uninterested in whether or not such differences are genetic or related to socialization. It is enough that the differences exist. These differences become stronger over time and influence one's entire world-view and life. For those who are just thinking about this idea for the first time, I recommend that you find a little book called *The Yellow Wallpaper* (Gilman, 1973). Written over 100 years ago, it gives a picture of one woman's view of her world and, at the same time, the picture of that same world from her husband's viewpoint. Both pictures impress the reader with their accuracy—and they are dramatically different.

Feminist scholars work in many areas and almost all of them are outside mathematics education. Some are trying to interpret a basic discipline of science (such as biology or history) from a female, rather than a male, point of view. They argue that almost all scholarship, including the development of what is called science and mathematics, has been done by men from a masculine viewpoint, utilizing values that are shared by men, but not by women. Those major bodies of knowledge that appear to be value-free and to report universal truths are in reality based on masculine values and perceptions. Since males' roles and spheres in the world have been so different from females' roles and spheres (Greene, 1984), these bodies of knowledge do not reflect 50 percent of human beings and thus are incomplete and inaccurate. Jim Schuerich (1992) has suggested that a feminist science is better than a value-free science. To support this, he draws from Charol Shakeshaft's (1987) work on educational administration and Carol Gilligan's (1982) work on the development of moral judgment. Each of them has demonstrated quite conclusively that research on male-only populations has produced results that were not only incomplete but *wrong*. Similar approaches to history and literature have resulted in deeper, richer understanding.

The idea of masculine-based interpretations in areas such as history or literature, and even in medical science, is not too difficult to illustrate nor even to accept. After all, until recently history didn't bother to include many females except for those few who happened to be queens or were burned at the stake. For example, how many even knew who Sacajawea was or her contributions to the opening of our American West until very recently. (Did you know that was her image on the new dollar coin?) Many conclusions in medical research have been based solely on male subjects; their inaccuracy is easy to illustrate. History has been presented as if most of our ancestors were male and as if important things in the public arena happened predominantly because of and to males. The use of male names by female writers in order that their writing be accepted, or even published, is commonly known.

Does the prevalence of the idea of a masculine or feminine world-view apply to what mathematics is and if so, how? Can mathematics be seen as masculine or feminine? Is not mathematics a logical, value-free field? The idea of a masculine or feminine mathematics is difficult to accept and to understand, even for many who have been concerned about gender and mathematics. But a few people are working to explicate what a gendered mathematics might be—in particular, Suzanne Damarin (1995), and Zelda Isaacson (1986), are struggling to define what a feminist approach to the study of mathematics education might be.

One way to approach the problem of a gendered mathematics is not to look at the subject, but to examine the way that people think and learn within the subject. The work of Belenky and her colleagues (Belenky, Clinchy, Goldberger, & Tarule, 1986) in identifying women's ways of thinking and knowing is provocative as we consider these questions. Do females learn math differently than do males? Should we develop special instructional programs for females? It is beyond the scope of this paper to explicate these ideas, but we should consider this. Earlier I alluded to the idea that most mathematics classrooms appear to be organized to be more appropriate for boys than for girls and even quoted some research that supports that idea. Others have interpreted discussions arising from the belief about a female world-view

and applied the ideas to describing female-friendly instruction. Such instruction usually includes such things as the greater inclusion of cooperation rather than of competition in classrooms, small group rather than individual work, more communication, and/or more socially relevant mathematics. Others have argued for single-sex schools oriented to the mathematics instruction of females. Running through these suggestions, it seems to me, is a basic belief that females learn differently and perform differently in mathematics than do males. This belief is dramatically different than the belief of a universal way of human thinking espoused by the cognitive scientists.

Another theme that informs many of the feminist perspectives is the necessity for females' voices to be heard (Campbell & Greenberg, 1993). To scholars with this conviction, it is not enough that scholars identify important research questions that are studied objectively using a positivist approach. Females must have a hand in the identification of the questions and the research itself so that females' life experiences can take center stage. Thus, the world can, and will be, interpreted from a female perspective. Scholars who work within this belief system often use women subjects as co-investigators, have females reporting their own experiences, and use females as the main subjects under investigation who also help to interpret results. There are not many of these studies available currently in mathematics education, but I predict that we shall see increasing numbers of them as the importance of female voices is recognized. Those that are available indicate that females often have a very negative view of mathematics, how it is taught, and mathematicians.

It is too early to be able to assess the impact that studies using feminist methodologies will have on our understanding of the relationship between gender and mathematics, both the identification of the problem and its solutions. But, it appears logical to me that as I try to interpret the problem from a feminist standpoint, the focus used in my earlier work changes. I do not interpret the challenges related to gender and mathematics as involving *problems* of females (e.g. females are deficient because they are less confident, don't believe mathematics is useful, lack spatial skills, etc.) or design interventions based on the masculine world-view of changing the females so they are equal to males. Instead, I begin to look at how a male view of mathematics has been destructive to females. I begin to articulate the problem that lies in our current views of mathematics and its teaching. I am coming to believe that females have recognized that mathematics, as currently taught and learned, restricts their lives rather than enriches them.

I must say at this point that the current reform movement has strong feminine overtones (and that is an anathema to many people). But the emphasis on students' views (their thinking), communication, social relevant mathematics could have come straight from many feminist scholars.

Whatever our own value position about feminism and mathematics, I believe that we need to examine carefully how feminist perspectives can add enriched understanding to our knowledge of mathematics education. And, indeed, we should be open to the possibility that we have been so enculturated by the masculine-dominated society we live in that our belief about the gender neutrality of mathematics as a discipline may be wrong or, at the very least, incomplete. Perhaps we have been asking the wrong questions as we have studied gender and mathematics. Could there be a better set of questions, studied from feminist perspectives, that would help us understand gender issues in mathematics? What would a feminist mathematics be? Is there a female way of thinking about mathematics? Would mathematics education, organized from a feminist perspective, be different from the mathematics education we currently have? Suzanne Damarin (1995) stated that we need to "create a radical reorganization of the ways that we think about and interpret issues and studies of gender and mathematics." Many scholars believe that only as we do this will we be able to understand gender issues in mathematics. Perhaps my beginning to believe that the decision by females not to learn mathematics or enter mathematics-related careers because mathematics has not offered them a life they wish to lead is an indication that my old view about learning and teaching mathematics, as well as about gender and mathematics, was immature and incomplete. An examination of what the female voices in the new research are saying will help me—and perhaps others—to understand teaching, learning, gender, and mathematics better.

What Do I Know?

The Complexity

Throughout this paper, I have been expounding on the complexity of dealing with gender and mathematics. Nothing appears to be simple and listing what I really know is difficult. That females participate in mathematics-related careers less than do males is one of the few accepted facts. That differences exist in the learning of mathematics seems clear also, although many scholars believe either that the differences are diminishing or that any differences that exist are unimportant. Females appear to hold more negative values about mathematics and their own relationship to mathematics than do males but there is some evidence that these differences are decreasing (Forgasz, Leder, & Vale, 1999). But I caution everyone about such simplistic statements. What mathematics is being measured in tests where gender differences are either shown or not shown? How was the information about values obtained? Were females' voices a part of the data gathering procedures? Too often research dealing with these issues provides an incomplete picture at best and only helps to perpetuate the belief that females are somehow inadequate in relation to learning and doing mathematics.

Dilemmas for Practice

Two of my colleagues and I have identified some dilemmas that we face as we interpret a variety of kinds of research and reform recommendations that are appropriate for organizing classroom instruction (Ambrose, Levi, & Fennema, 1997). Many of these appear logically to apply equally to girls and boys. But a closer examination reveals that nothing to do with gender is simple.

Consider the reform recommendation that has to do with encouraging students to communicate their mathematics thinking by presenting their ideas and convincing peers of their correctness by arguing, questioning, and disagreeing. It is widely believed that those who enter into this kind of debate will learn better. But will girls enter into this kind of communication as willing as do boys? Many teachers have reported informally that girls will not for a variety of reasons. Perhaps this even helps to explain some of the gender differences we reported in the CGI study. Will boys tend to dominate such discussions and not listen as well as girls?

Another major reform recommendation has to do with the use of technology in the classroom. Others at this conference have discussed this as it relates to girls and boys. It is clear that boys have more experience with technological toys than do girls. Does this reflect interest? Does this mean boys have more knowledge? How do teachers take these ideas into consideration?

The *Standards* recommend that mathematics be situated in problem-solving contexts that are socially relevant. Unfortunately many textbooks and teachers are more aware of contexts that are from male dominated fields such as projectiles for parabolic equations, or sports for statistics. One interesting study that I did not review earlier suggests that gender differences in problem solving skills were eliminated when girls were familiar with the context in which the problem was situated (Marshall, 1984). But will boys willingly participate in problems from female-dominated fields?

Should classrooms be competitively organized or organized around cooperative activities? Certainly the most visible reward in most mathematics classrooms is grades that are highly competitive. The Fennema-Peterson studies quoted earlier suggest that young boys learned better in a competitive situation while young girls learned better in a cooperative situation. Is that finding true for older students? Is the solution to have single-sex classrooms? And would the experience we have had with black/white schools be repeated and females classrooms become inevitably less adequate?

Thoughtful, reasonable practitioners can probably create solutions to each of the dilemmas presented. But it is the role of researchers to help them identify the potential problems that may exist and aid them in evaluating any solutions that are created.

What Do I Wish Was Known?

The Future Contributions of Research

Research into gender and mathematics must continue. We should continue to monitor the best we can learning, attitudes, and participation in mathematics. In addition, we need to develop new paradigms of research that will provide insight into why gender differences occur. In other words, gender as a critical variable must enter the mainstream of mathematics education research. It is insufficient to say and to believe that the study of gender differences can be left to those who are specifically interested in gender. *That is not just nor fair!* Aren't we all interested in how ALL learn mathematics? And ALL includes that 50% of the student body who happen to be female. Fairness and justice demand that ALL researchers be concerned with ALL the students even when results are obtained that can not be easily interpreted and understood.

Specifically, we need to continue the study of gender in relation to mental processing of both students and teachers. As research on teachers continues to mature and improve, we must include gender as a variable. We probably cannot study how the sex of the teacher influences instruction because of the limitations imposed by the number of male teachers available. However, we can study teachers' beliefs and knowledge about girls and boys and the impact that teachers' cognition has on instructional decisions for both girls and boys.

Classrooms that reflect the various demands for reform are beginning to become more and more prevalent. But are they equally effective for boys and girls? The CGI study discussed earlier provides some evidence that just reforming classrooms without specific attention to traditionally under achieving groups is insufficient to achieve equity. The learning that results from these reformed classrooms needs to be carefully monitored. Perhaps as we do this, we will begin to develop an image of what equitable mathematics education is.

Values and Research

Personal values dominate the doing and interpreting of research in gender and mathematics. I think I became an educational researcher because I believed that I would discover TRUTH. That has not happened and I believe that if truth can be found from educational research, it is not in the area of gender and mathematics. But, research has deepened our knowledge about gender and mathematics and the many, many studies about gender have provided some insight into the inequities that have existed and that has led to heightened awareness of things that need to be changed. But there are some questions about gender and mathematics for which research cannot provide the answers. Is mathematics really necessary for a life of value in the 21st century? This is a heretical question coming from a mathematics educator but one that needs to be addressed. It appears to me that I may have been attaching the worthwhileness of an individual to whether or not she or he learns mathematics. Now, in fairness to me, I have spent my professional career in trying to assist traditionally underachieving groups to learn mathematics, but Nel Noddings (1998) has led me to examine my own beliefs in this regard. She had the courage to raise a very difficult question about major differences between boys and girls and mathematics as she addressed the differences found in the CGI study she relates the results to the possibility "that girls are just less interested in math than (are) boys."

If it were true that girls are less interested than boys in math, *so what?* What would follow? Clearly, we still could not judge the next female or male who walks into out classroom on the basis of this generalization. The next female may be Hypatia reincarnated and the next male Forrest Gump. Further, the generalization in itself doesn't tell us what to do. . . . A positive answer to the question about gender differences in interest in math might lead to further exploration of an idea that repels many of us, i.e., the question of genetic differences. . . . But the genetic argument does not seem particularly helpful to us as educators and launching the argument about interest in math would enable

us to examine the question of gender differences in a way that might be helpful. . . . Why do we see it as a problem if young women are less interested than young men in mathematics? Why *don't* we see it as a problem if young men are less interested than young women in early childhood education, nursing, elementary teaching, and full-time parenting? The easy answer to the issue posed in this fashion is that proficiency in mathematics opens the doors to professional success and financial well-being. There's no money in the other activities. But consider what is being valorized. Why is there so little financial compensation and prestige in fields traditionally associated with women? . . . Do we approve of a social structure that values competence in mathematics over competence in child-care? . . . No student's self-worth should depend on her or his interest or capability in mathematics, and we should not endorse the propaganda that mathematics is essential in almost all worthwhile occupations. . . . We must explore the unpleasant possibility that many girls do not *want* to be part of the math crowd because many of its members seem socially inept or aloof. (pp. 17-18)

I shall end with some personal soul searching that I have been engaged in. There are no *right* answers but perhaps we should consider the following. Is it possible that I, and others who have been doing work related to gender and mathematics, have been doing a major injustice to females by pursuing issues related to gender and mathematics? Are we just making the chosen roles of females in society (that often don't involve mathematics) less important, less adequate, or of less value than the chosen roles of males (that often include mathematics)? Is it critical for everyone to learn mathematics? Are those who learn mathematics at lower levels of less value than those who learn at higher levels?

Research on gender and mathematics has provided a powerful scientific discourse during the past 3 decades. The entire educational community—composed of practitioners, researchers, and policymakers—need to continue to engage in this discourse about and to explore ways to deepen our understanding of what equity is and how it can be achieved. It is in discourse about philosophical questions as well as research questions that our understanding of gender and mathematics will grow.

Endnote

Some of this paper was excerpted from:

Fennema, E. (1996). Mathematics, gender and research. In G. Hanna (Ed.), *Towards gender equity in mathematics education* (pp. 9-26). Amsterdam: Kluwer.

Fennema, E., & Carpenter, T. P. (1998). New perspectives on gender differences in mathematics: An introduction and a reprise. *Educational Researcher*, 4(11), 19-22.

References

- Adetula, L. O. (1989). Solution of simple word problems by Nigerian children: Language and schooling factors. *Journal for Research in Mathematics Education*, 20, 489-497.
- Ambrose, R., Levi, L., & Fennema, E. (1997). The complexity of teaching for gender equity. In J. Trentacost & M. J. Kenney (Eds.), *Multicultural and gender equity in the mathematics classroom: The gift of diversity* (pp. 236-242). 1997 Yearbook. Reston, VA: National Council of Teachers of Mathematics.
- Anno nymous. (Manuscript in preparation). Citation classic. Current Contents.
- Belenky, M., Clinchy, B., Goldberger, N., & Tarule, J. (1986). *Women's ways of knowing: The development of self, voice, and mind*. New York: Basic Books.
- Brown, C., & Borko, H. (1992). Becoming a mathematics teacher. In D. A. Grouws, (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan.
- Campbell, P. B., & Greenberg, S. (1993). Equity issues in educational research methods. In S. K. Bilken & D. Pollard (Eds.), *Gender and education: Ninety-second yearbook of the National Society for the Study of Education*. Chicago: University of Chicago Press.
- Campbell, P. B., & Storo, J. (1996). *Making it happen: Pizza parties, chemistry goddesses and other strategies that work for girls and others*. Newton, MA: Education Development Center.
- Campbell, P. B., & Wahl, E. (In press). Of two minds: Single sex. Co-education and the search for gender equity. *New York Law School Journal of Human Rights*.
- Carey, D. A., Fennema, E., Carpenter, T. P., & Franke, M. L. (1995). Equity and mathematics education. In W. Secada, E. Fennema, & L. Byrd (Eds.), *New directions for mathematics education*. New York: Teachers College Press.
- Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (1999). *Children's mathematics: Cognitively guided instruction*. Portsmouth, NH: Heinemann.
- Carr, M., & Jessup, D. L. (1997). Gender differences in first grade mathematics strategy use: Social and metacognitive influences. *Journal of Educational Psychology*, 98(2), 318-328.
- Damarin, S. K. (1995). Gender and mathematics from a feminist standpoint. In W. G. Secada, E. Fennema, & L. B. Adajian (Eds.), *New directions for equity in mathematics education* (pp. 242-257). New York: Cambridge University Press.
- Eccles, J. S., & Blumenfeld, P. (1985). Classroom experiences and student gender: Are there differences and do they matter? In L. C. Wilkinson & C. B. Marrett (Eds.), *Gender influences in classroom interaction*. New York: Academic Press.
- Fennema, E. (1996). Mathematics, gender, and research. In G. Hanna (Ed.), *Towards gender equity in mathematics education* (pp. 9-26). Amsterdam: Kluwer.
- Fennema, E., & Carpenter, T. P. (1998). New perspectives on gender differences in mathematics: An introduction and a reprise. *Educational Researcher*, 27(5), 4-11, 19-22.

Fennema, E., & Leder, G. (Eds.). (1990). *Mathematics and gender: Influences on teachers and students*. New York: Teachers College Press.

Fennema, E., & Peterson, P. L. (1986). Teacher-student interactions and sex-related differences in learning mathematics. *Teaching and Teacher Education*, 2(1), 19-42.

Fennema, E., Peterson, P. L., Carpenter, T. P., & Lubinski, C. A. (1990). Teachers' attributions and beliefs about girls, boys, and mathematics. *Educational Studies in Mathematics*, 21(1), 55-65.

Fennema, E., & Sherman, J. (1977). Sex-related differences in mathematics achievement, spatial visualization, and affective factors. *American Educational Research Journal*, 14(1), 51-71.

Fennema, E., & Sherman, J. (1978). Sex-related differences in mathematics achievement and related factors: A further study. *Journal for Research in Mathematics Education*, 9(3), 189-203.

Fennema, E., & Tartre, L. (1985). The use of spatial visualization in mathematics by boys and girls. *Journal of Research in Mathematics Education*, 16(3), 184-206.

Fennema, E., Wolleat, P., Becker, A., & Pedro, J. D. (1980). *Multiplying options and subtracting biases*. (An intervention program composed of four videotapes and facilitators' guides). Reston, VA: National Council of Teachers of Mathematics.

Forgasz, H. J., Leder, G. C., & Vale, C. (1999). Gender and mathematics: Changing perspectives. In K. D. Owens & J. Mcousley (Eds.), *Research in mathematics education in Australia 1996-1999*. Sydney, Australia: MERGA.

Gallagher, A. M., & DeLisi, R. (1994). Gender differences in scholastic aptitude tests—mathematics problem solving among high-ability students. *Journal of Educational Psychology*, 86, 204-211.

Gilligan, C. (1982). *In a different voice: Psychological theory and women's development*. Cambridge, MA: Harvard University Press.

Gilman, C. P. (1973). *The Yellow Wallpaper*. (Afterword by E. R. Hedges). New York: Feminist Press. (Original work published 1899.)

Greene, M. (1984). The impacts of irrelevance: Women in the history of American education. In E. Fennema & M. J. Ayer (Eds.), *Women and education: Equity or equality?* Berkley, CA: McCutchan.

Hanna, G. (1989). Mathematics achievement of girls and boys in grade eight: Results from twenty countries. *Educational Studies in Mathematics*, 20(2), 225-232.

Hyde, J. S., Fennema, E., & Lamon, S. J. (1990). Gender differences in mathematics performance. *Psychological Bulletin*, 139-155.

Hyde, J. S., Fennema, E., Ryan, M., & Frost, L. A. (1990). Gender differences in mathematics attitude and affect: A meta-analysis. *Psychology of Women Quarterly*, 14, 299-324.

Hyde, J. S., & Jaffee, S. (1998). Perspectives from social and feminist psychology. *Educational Researcher*, 27(5), 14-16.

Isaacson, Z. (1986). Freedom and girls' education: A philosophical discussion with particular reference to mathematics. In L. Burton (Ed.), *Girls into maths can go*. London: Holt, Rinehart & Winston.

- Jacobs, J. E., & Eccles, J. S. (1985). Gender differences in math ability: The impact of media reports on parents. *Educational Research*, 14(3), 20-25.
- Koehler, M. S. (1990). Classrooms, teachers, and gender differences in mathematics. In E. Fennema & G. Leder (Eds.), *Mathematics and gender*. New York: Teachers College Press.
- Lagemann, E. L., & Shulman, L.S. (1999). *Issues in education research: Problems and possibilities*. San Francisco, CA: Jossey-Bass.
- Leder, G. C. (1992). Mathematics and gender: Changing perspectives. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan.
- Leder, G. C, Forgasz, H. J., & Solar, C. (1996). Research and intervention programs in mathematics education: A gendered issue. In A. J. Bishop et al. (Eds.), *International handbook of mathematics education* (pp. 945-985). Netherlands: Kluwer Academic Publishers.
- Maccoby, E. E., & Jacklin, C. N. (1974). *The psychology of sex differences*. Stanford: Stanford University Press.
- Marshall, S. P. (1984). Sex differences in children's mathematics achievement: Solving computations and story problems. *Journal of Education Psychology*, 76(2), 194-204.
- National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.
- Noddings, N. (1998). Perspectives from feminist philosophy. *Educational Researcher*, 27(5), 17-18.
- Olivier, A., Murray, H., & Human, P. (1990). Building your children's mathematical knowledge. In G. Booker, P. Cobb, & T. N. Mendicuti (Eds.), *Proceedings of the Fourteenth International Conference for the Psychology of Mathematics Education* (Vol. 3, pp. 297-304). Mexico.
- Peterson, P. L., & Fennema, E. (1985). Effective teaching, student engagement in classroom activities, and sex-related differences in learning mathematics. *American Educational Research Journal*, 22(3), 309-335.
- Reys, R. E., Rybolt, J. F., Bestgen, B. J., & Wyatt, J. W. (1982). Processes used by good computational estimators. *Journal for Research in Mathematics Education*, 13, 183-201.
- Schuerich, J. (1992). Methodological implications of feminist and poststructuralist views of science. *The National Center for Science Teaching and Learning Monograph Series*, 4.
- Secada, W. G. (1989). Educational equity versus equality of education: An alternative conception. In W. G. Secada (Ed.), *Equity in education* (pp. 68-88). London: Falmer Press.
- Secada, W. G. (1991). Degrees of bilingualism and arithmetic problem solving in Hispanic first graders. *Elementary School Journal*, 52, 306-316.
- Secada, W. G. (1992). Race, ethnicity, social class, language, and achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan.
- Shakeshaft, C. (1987). *Women in educational administration*. Newbury Park, CA: Sage.
- Sowder, J. T. (1998). Perspectives from mathematics education. *Educational Researcher*, 27(5), 12-13.

Stanley, J., & Benbow, C. (1980). Sex differences in mathematical ability: Fact or artifact. *Science*, 210, 1262-1264.

Walberg, H., & Haertel, G. (1992). Educational psychology's first century. *Journal of Educational Psychology*, 84(1), 6-19.

Weisbeck, L. (1992). *Teachers' thoughts about children during mathematics instruction*. Unpublished doctoral dissertation, University of Wisconsin-Madison.